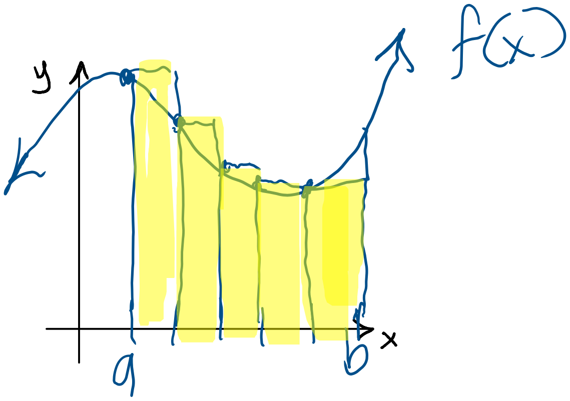
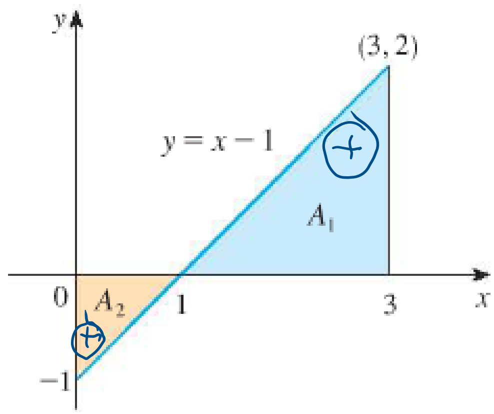
sometimes slices are over- AND under-estimations:



Recall: If area lies **BELOW** *x*-axis, that region is **negative** (subtracted).



Total area of shaded regions: $A_1 - A_2$

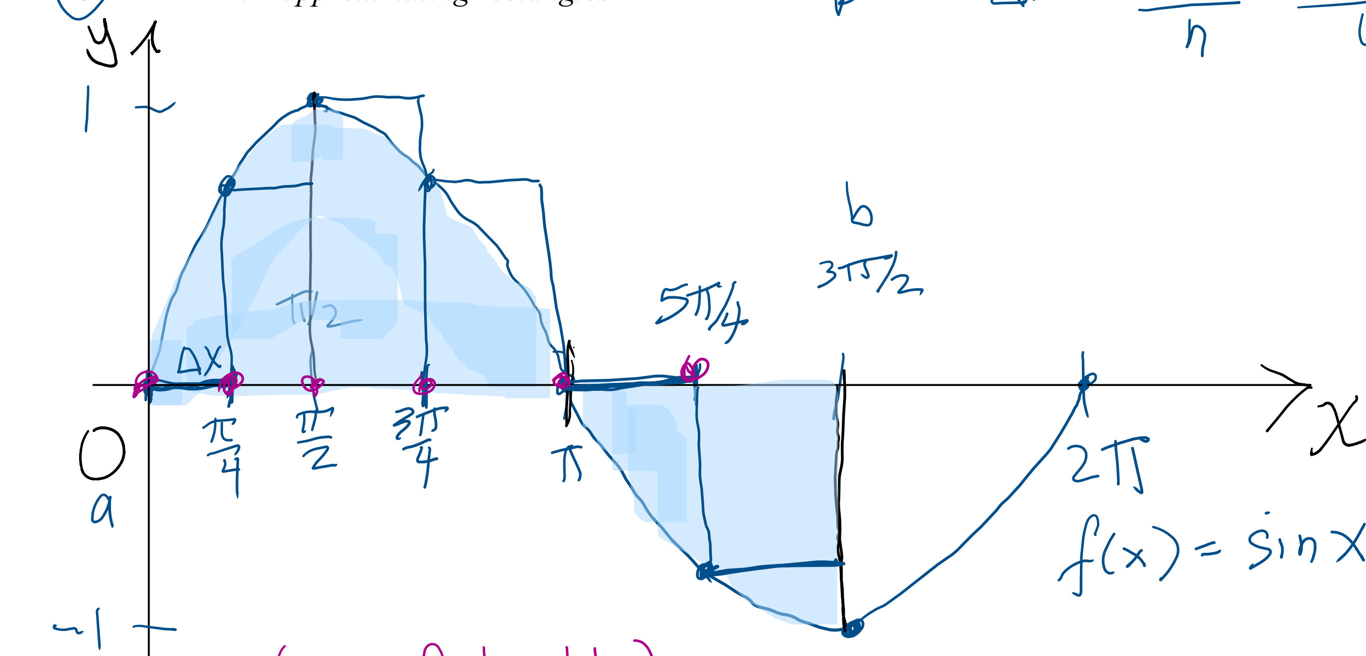
ex. SET UP the area under $f(x) = \sin x$ from $x = 0$ to $x = \frac{3\pi}{2}$ *using:*

$f(0) = \sin 0$ a b

n 6 *LEFT-HAND approximating rectangles*

$\Delta x = \frac{b-a}{n}$ $\frac{\frac{3\pi}{2} - 0}{6} = \frac{3\pi}{2} \cdot \frac{1}{6}$

$\Delta x = \frac{\pi}{4}$

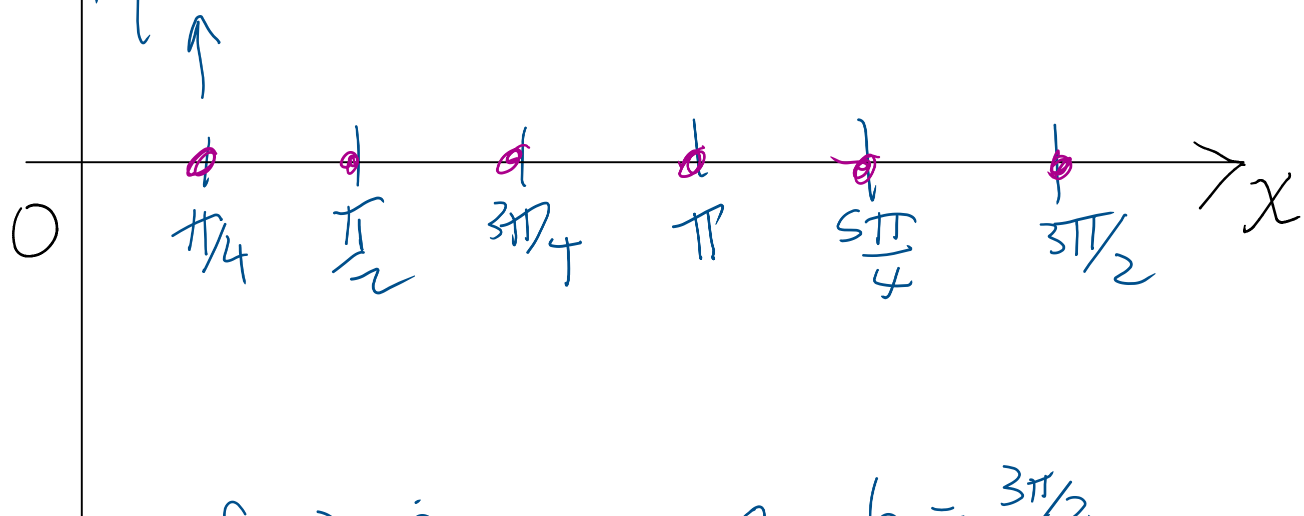


Δx (sum of heights)

$L_6 = \frac{\pi}{4}\left(\sin 0 + \sin\frac{\pi}{4} + \sin\frac{\pi}{2} + \sin\frac{3\pi}{4} + \sin\pi + \sin\frac{5\pi}{4}\right)$

6 RIGHT-HAND approximating rectangles

$$R_6 = \frac{\pi}{4}\left(\sin\frac{\pi}{4} + \sin\frac{\pi}{2} + \sin\frac{3\pi}{4} + \sin\pi + \sin\frac{5\pi}{4} + \sin\frac{3\pi}{2}\right)$$

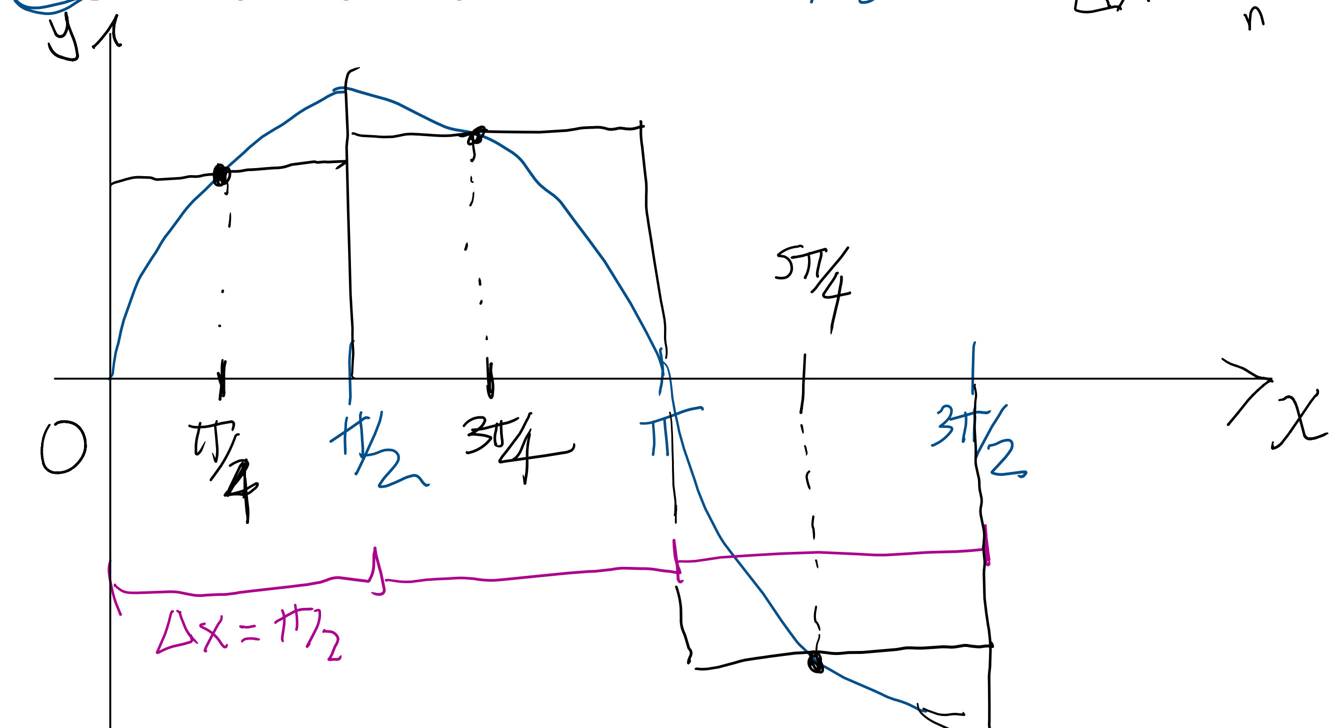


given $f(x) = \sin x$ $a = 0$ $b = 3\pi/2$

Let's use a different method to determine the height of the slices: MIDPOINT RULE

3 approximating rectangles using MIDPOINTS M_3

$$\Delta x = \frac{b-a}{n} = \frac{\frac{3\pi}{2}}{3} = \frac{3\pi}{2}\cdot\frac{1}{3} = \frac{\pi}{2}$$

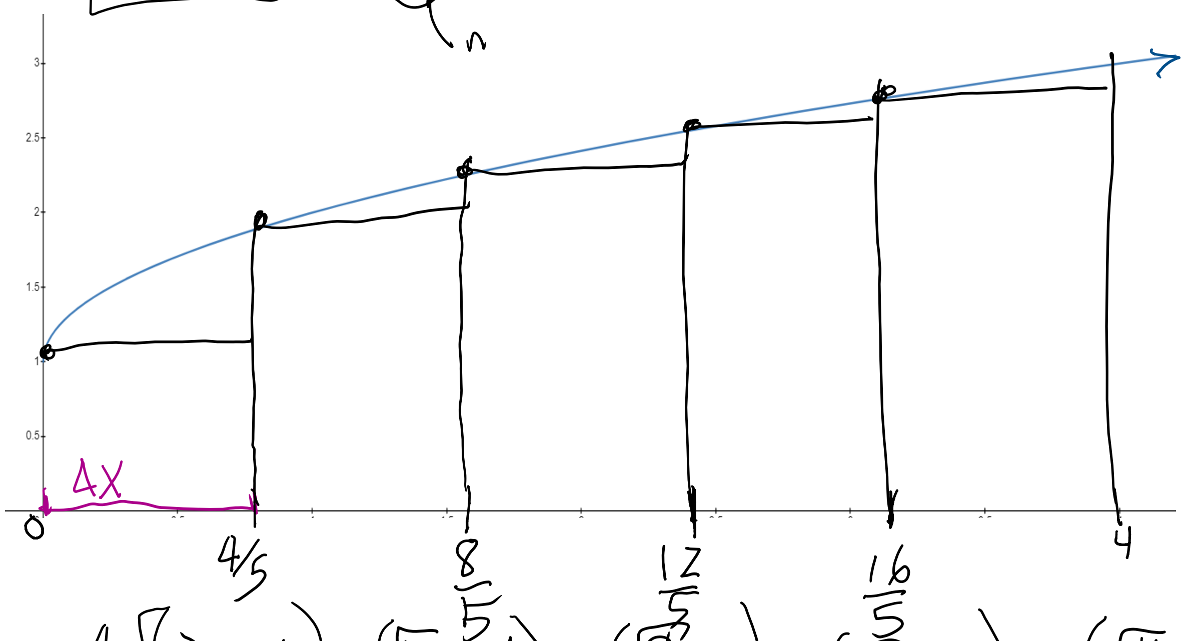


$$M_3 = \frac{\pi}{2}\left(\sin\frac{\pi}{4} + \sin\frac{3\pi}{4} + \sin\frac{5\pi}{4}\right)$$

$\Delta x = \frac{b-a}{n} = \frac{4-0}{5} = \frac{4}{5}$

Given $f(x)=\sqrt{x}+1$, SET UP L_5 between $x=0$ and $x=4$.

n



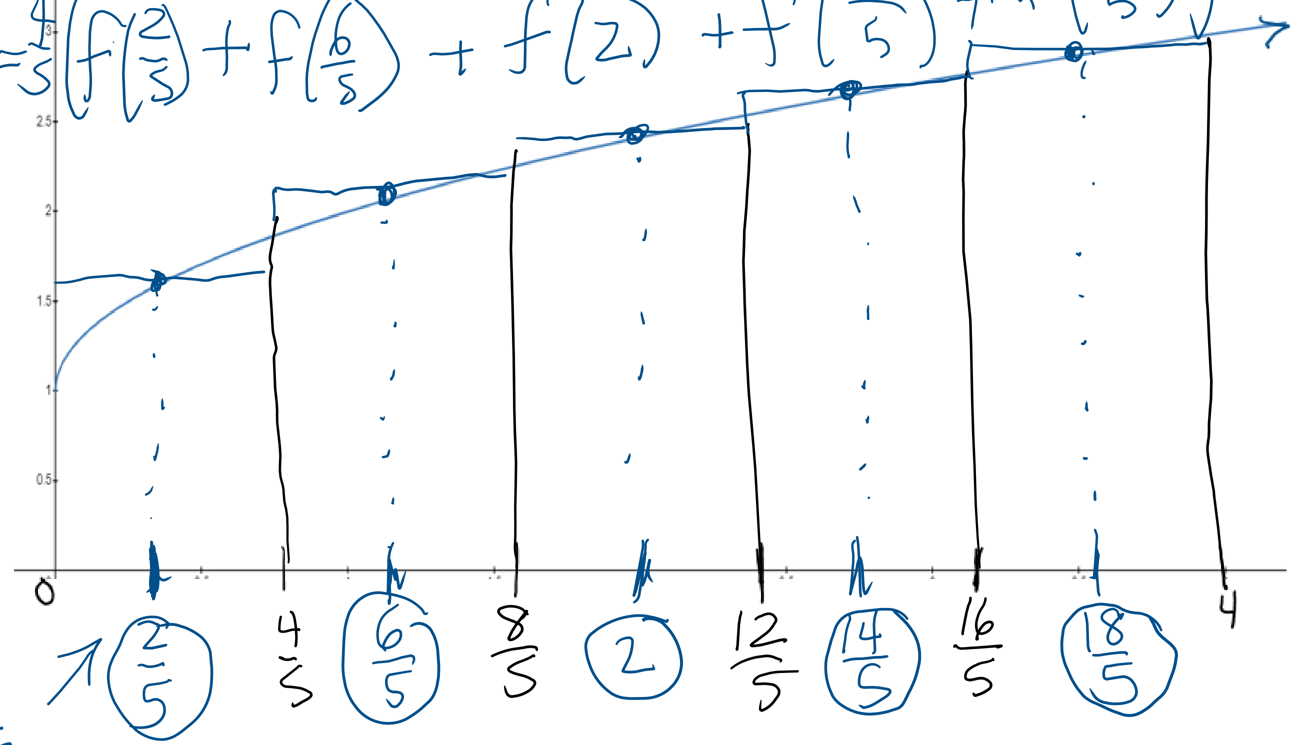
check $\frac{20}{5} = 4$ ✓

$$L_5 = \frac{4}{5}\left[(0+1)+\left(\sqrt{\frac{4}{5}}+1\right)+\left(\sqrt{\frac{8}{5}}+1\right)+\left(\sqrt{\frac{12}{5}}+1\right)+\left(\sqrt{\frac{16}{5}}+1\right)\right]$$

Given $f(x)=\sqrt{x}+1$, SET UP M_5 between $x=0$ and $x=4$.

Δx

$$M_5 = \frac{4}{5}\left(f\left(\frac{2}{5}\right)+f\left(\frac{6}{5}\right)+f(2)+f\left(\frac{14}{5}\right)+f\left(\frac{18}{5}\right)\right)$$



$\frac{1}{2} \cdot \frac{4}{5}$

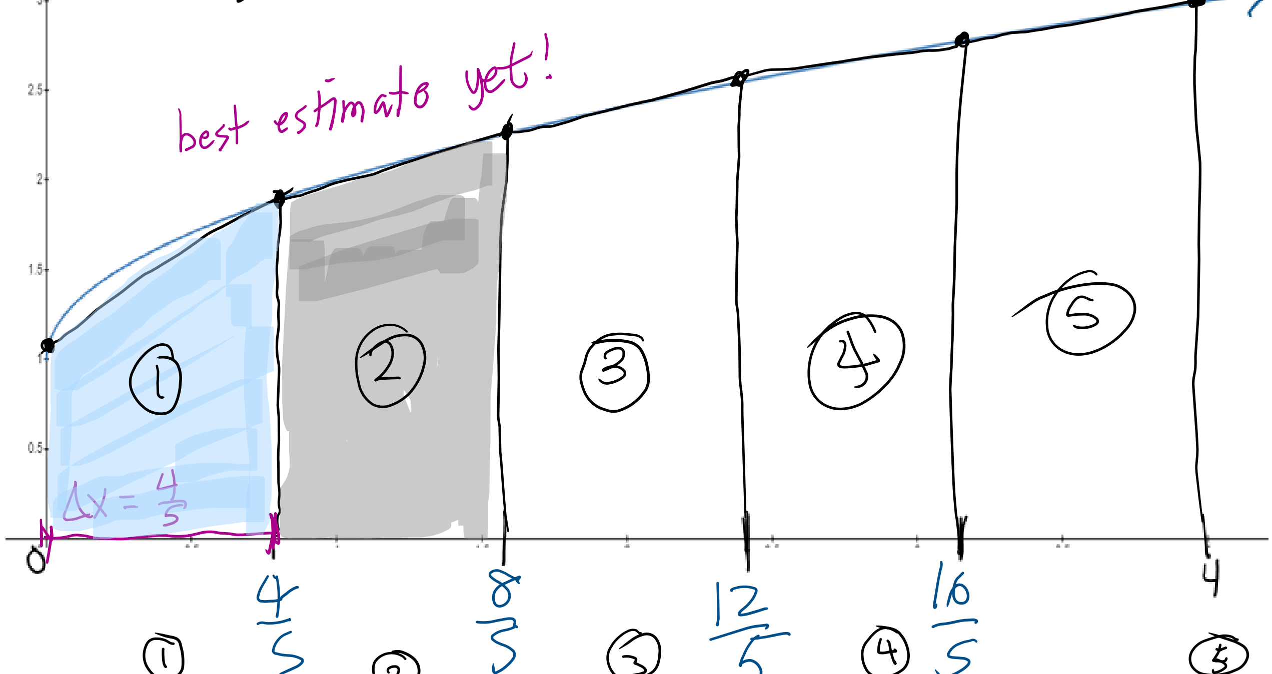
$\frac{x_1 + x_2}{2}$ = midpoint

One more way to determine heights: TRAPEZOIDAL RULE from $x=0$ to $x=4$

find T_5

to create slices, connect ordered pairs w straight lines

$f(x)=\sqrt{x}+1$



$$T_5 = \frac{1}{2}\cdot\frac{4}{5}\left(\overbrace{f(0)+f\left(\tfrac{4}{5}\right)}^{1}+\overbrace{f\left(\tfrac{4}{5}\right)+f\left(\tfrac{8}{5}\right)}^{2}+\overbrace{f\left(\tfrac{8}{5}\right)+f\left(\tfrac{12}{5}\right)}^{3}+\overbrace{f\left(\tfrac{12}{5}\right)+f\left(\tfrac{16}{5}\right)}^{4}+\overbrace{f\left(\tfrac{16}{5}\right)+f(4)}^{5}\right)$$

$$T_5 = \frac{1}{2}\cdot\Delta x\left(f(0)+2f\left(\tfrac{4}{5}\right)+2f\left(\tfrac{8}{5}\right)+2f\left(\tfrac{12}{5}\right)+2f\left(\tfrac{16}{5}\right)+f(4)\right)$$

i = iteration $\Rightarrow$ increase incrementally by 1

Sigma Notation

a condensed way of writing sums uses capital sigma: Σ

ex. Convert the sum $3+4+5+6+7$ into sigma notation.

$i=3 \quad i=4 \quad i=5 \quad i=6 \quad i=7$

$$\sum_{i=3}^{7} i = 3+4+5+6+7$$

$i=$ 3, 4, 5, 6, 7

ex. $\sum_{i=3}^{7} \frac{1}{i} = \frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}$

$i=1 \quad i=2 \quad i=3 \quad i=4$

ex. $\sum_{i=1}^{4} 2 = \underbrace{2+2+2+2} = \underbrace{2(4)} = \boxed{8}$

ex. $\sum_{i=1}^{n} 1 = \overbrace{1+1+\dots+1}^{\text{1 being add n times}} = 1\cdot n = \boxed{n}$

REVISIT setting up area estimate L_3 under $f(x)=x^2$ from $x=1$ to $x=4$ using *sigma notation*.

$f(i) = i^2$

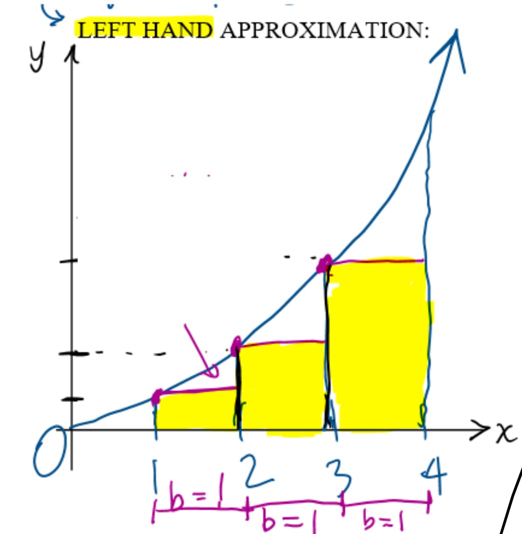
Δx

$= 1\,(f(1)+f(2)+f(3))$

$= 1 \sum_{i=1}^{3} f(i)$

$= \boxed{\sum_{i=1}^{3} i^2}$

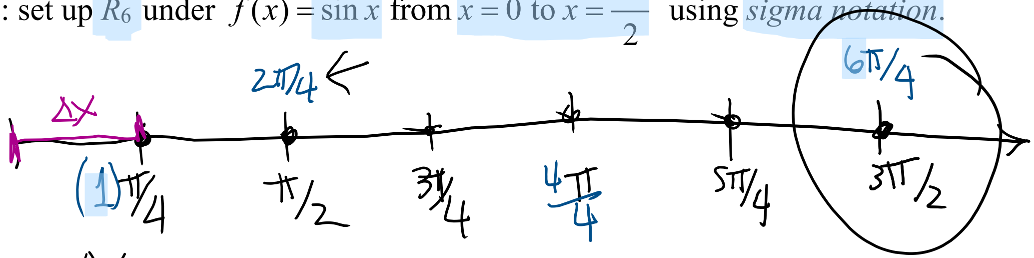
LEFT HAND APPROXIMATION:



$L_3 = bh_1 + bh_2 + bh_3$

$= 1f(1) + 1f(2) + 1f(3)$

REVISIT: set up R_6 under $f(x) = \sin x$ from $x = 0$ to $x = \frac{3\pi}{2}$ using *sigma notation*.



$$R_6 = \overset{\Delta x}{\frac{\pi}{4}} \sum_{i=1}^{6} \sin\left(\frac{\pi}{4}\cdot i\right)$$

EXPAND TO CHECK

$$= \frac{\pi}{4}\left(\overset{i=1}{\sin\frac{\pi}{4}(1)} + \overset{i=2}{\sin\left(\frac{\pi}{4}\cdot 2\right)} + \overset{i=3}{\sin\left(\frac{\pi}{4}\cdot 3\right)} + \cdots + \overset{i=6}{\underset{\downarrow\ \sin\frac{3\pi}{2}}{\sin\left(\frac{\pi}{4}\cdot 6\right)}}\right)$$

Given $f(x) = \sqrt{x} + 1$, express R_5 between $x = 0$ and $x = 4$ using *sigma notation.*